3

Design of Shallow Foundations

by

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3.1 Introduction

A foundation is a structural element that is expected to transfer a load from a structure to the ground safely. The two major classes of foundations are shallow foundations and deep foundations. A shallow foundation transfers the entire load at a relatively shallow depth. A common understanding is that the depth of a shallow foundation \((D_f)\) must be less than the breadth \((B)\). Breadth is the shorter of the two plan dimensions. Shallow foundations include pad footings, strip (or wall) footings, combined footings, and mat foundations, shown in Figure 3.1. Deep foundations have a greater depth than breadth and include piles, pile groups, and piers, which are discussed in Chapter 4. A typical building can apply 10–15 kPa per floor, depending on the column spacing, type of structure, and number of floors.

Shallow foundations generally are designed to satisfy two criteria: bearing capacity and settlement. The bearing capacity criterion ensures that there is adequate safety against possible bearing capacity failure within the underlying soil. This is done through provision of an adequate factor of safety of about 3. In other words, shallow foundations are designed to carry a working load of one-third of the failure load. For raft foundations, a safety factor of 1.7–2.5 is recommended (Bowles 1996). The settlement criterion ensures that settlement is within acceptable limits. For example, pad and strip footings in granular soils generally are designed to settle less than 25 mm.

3.2 Stresses beneath Loaded Areas

In particular for computing settlement of footings, it is necessary to be able to estimate the stress increase at a specific depth due to the foundation loading. The theories developed for computing settlement often assume the soil to be a homogeneous, isotropic, weightless elastic continuum.
3.2.1 Point and Line Loads

Boussinesq (1885) showed that in a homogeneous, isotropic elastic half-space, the vertical stress increase ($\Delta\sigma_v$) at a point within the medium, due to a point load ($Q$) applied at the surface (see Figure 3.2), is given by

$$\Delta\sigma_v = \frac{3Q}{2\pi z^2} \left[ \frac{1}{1 + \left(\frac{x}{z}\right)^2} \right]^{5/2} (3.1)$$

where $z$ and $x$ are the vertical and horizontal distance, respectively, to the point of interest from the applied load.

Westergaard (1938) did similar research, assuming the soil to be reinforced by closely spaced rigid sheets of infinitesimal thicknesses, and proposed a slightly different equation:

$$\Delta\sigma_v = \frac{Q}{2\pi z^2} \sqrt{\frac{1 - 2\nu}{2 - 2\nu}} \left[ \frac{1 - 2\nu}{2 - 2\nu} \left( \frac{1}{\frac{2}{2 - 2\nu}} \right) + \left( \frac{x}{z} \right)^2 \right]^{3/2} (3.2)$$

Westergaard’s equation models anisotropic sedimentary clays with several thin seams of sand lenses interbedded with the clays. The stresses computed from the Boussinesq equation generally are greater than those computed from the Westergaard equation. As it is conservative and simpler, the Boussinesq equation is more popular and will be used throughout this section.

If the point load is replaced by an infinitely long line load in Figure 3.2, the vertical stress increase $\Delta\sigma_v$ is given by:

$$\Delta\sigma_v = \frac{2Q}{\pi z} \left[ \frac{1}{1 + \left(\frac{x}{z}\right)^2} \right]^2 (3.3)$$

3.2.2 Uniform Rectangular Loads

The vertical stress increase at a depth $z$ beneath the corner of a uniform rectangular load (see Figure 3.3a) can be obtained by breaking the rectangular load into an infinite number of point loads ($dq = Qdx dy$) and integrating over the entire area. The vertical stress increase is given by

$$\Delta\sigma_v = \frac{2Q}{\pi z} \left[ \frac{1}{1 + \left(\frac{x}{z}\right)^2} \right]^2 (3.3)$$
\[ \Delta \sigma_v = Iq \]  

(3.4)

where \( q \) is the applied pressure and the influence factor \( I \) is given by:

\[
I = \frac{1}{4\pi} \left[ \left( \frac{2mn \sqrt{m^2 + n^2 + 1}}{m^2 + n^2 + m^2 n^2 + 1} \right) \left( \frac{m^2 + n^2 + 2}{m^2 + n^2 + 1} \right) + \tan^{-1} \left( \frac{2mn \sqrt{m^2 + n^2 + 1}}{m^2 + n^2 - m^2 n^2 + 1} \right) \right]  

(3.5)

Here \( m = B/z \) and \( n = L/z \). Variation of \( I \) with \( m \) and \( n \) is shown in Figure 3.3b. Using the equation or Figure 3.3b, the vertical stress increase at any point within the soil, under a uniformly loaded rectangular footing, can be found. This will require breaking up the loaded area into four rectangles and applying the principle of superposition. This can be extended to T-shaped or L-shaped areas as well.

At a depth \( z \), \( \Delta \sigma_v \) is the maximum directly below the center and decays with horizontal distance. Very often, the value of \( \Delta \sigma_v \) is estimated by assuming that the soil pressure applied at the footing level is distributed through a rectangular prism, with slopes of 2 (vertical):1 (horizontal) in both directions, as shown in Figure 3.4. Assuming the 2:1 spread in the load, the vertical stress at depth \( z \) below the footing becomes:
$$\Delta \sigma_v = \frac{Q}{(B + z)(L + z)}$$  \hspace{1cm} (3.6)$$

In the case of strip footings, Equation 3.6 becomes:

$$\Delta \sigma_v = \frac{Q}{B + z}$$  \hspace{1cm} (3.7)$$

### 3.2.3 Newmark’s Chart for Uniformly Loaded Irregular Areas

The vertical stress increase at depth $z$ below the center of a uniformly loaded circular footing of radius $r$ is given by:

$$\Delta \sigma_v = \left\{ 1 - \frac{1}{[(r/z)^2 + 1]^{3/2}} \right\} q$$  \hspace{1cm} (3.8)$$

The values of $r/z$ for $\Delta \sigma_v = 0.1q, 0.2q...1.0q$ are given in Table 3.1. Newmark (1942) developed the influence chart shown in Figure 3.5 using the values given in Table 3.1. Each block in the chart contributes an equal amount of vertical stress increase at any point directly below the center. This chart can be used to determine the vertical stress increase at depth $z$ directly below any point ($X$) within or outside a uniformly loaded irregular area.

The following steps are required for computing $\Delta \sigma_v$ at depth $z$ below $P$:

1. Redraw (better to use tracing paper) the plan of the loaded area to a scale where $z$ is equal to the scale length given in the diagram.
2. Place the plan on top of the influence chart such that the point of interest $P$ on the plan coincides with the center of the chart.

<table>
<thead>
<tr>
<th>$\Delta \sigma_v / q$</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r/z$</td>
<td>0.270</td>
<td>0.401</td>
<td>0.518</td>
<td>0.637</td>
<td>0.766</td>
<td>0.918</td>
<td>1.110</td>
<td>1.387</td>
<td>1.908</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>
3. Count the number of blocks (say, \( n \)) covered by the loaded area (include fractions of the blocks).

4. Compute \( \Delta \sigma_v \) as \( \Delta \sigma_v = Iq \), where \( I \) is the influence value for Newmark’s chart. For the one in Figure 3.5, where there are 200 blocks, \( I = 1/120 = 0.00833 \).

### 3.3 Bearing Capacity of Shallow Foundations

Several researchers have studied bearing capacity of shallow foundations, analytically and using model tests in laboratories. Let’s look at some historical developments and three of the major bearing capacity equations with corresponding correction factors.

Typical pressure-settlement plots in different types of soils are shown in Figure 3.6. Three different failure mechanisms, namely general shear, local shear, and punching shear, were recognized by researchers. General shear failure is the most common mode of failure, and it occurs in firm ground, including dense granular soils and stiff clays, where the failure load is well defined (see Figure 3.6a). Here, the shear resistance is fully developed along the entire
failure surface that extends to the ground level, and a clearly formed heave appears at the
ground level near the footing. The other extreme is punching shear failure, which occurs in
weak, compressible soils such as very loose sands, where the failure surface does not extend to
the ground level and the failure load is not well defined, with no noticeable heave at the ground
level (Figure 3.6c). In between these two modes, there is local shear failure (Figure 3.6b), which
occurs in soils of intermediate compressibility such as medium-dense sands, where only slight
heave occurs at the ground level near the footing.

In reality, the ground conditions are always improved through compaction before placing
the footing. For shallow foundations in granular soils with $D_r > 70\%$ and stiff clays, the failure
will occur in the general shear mode (Vesic 1973). Therefore, it is reasonable to assume that
the general shear failure mode applies in most situations.

From bearing capacity considerations, the allowable bearing capacity ($q_{all}$) is defined as

$$q_{all} = \frac{q_{ult}}{F}$$  \hspace{1cm} (3.9)

where $q_{ult}$ is the ultimate bearing capacity, which is the average contact pressure at the soil-
footing interface when the bearing capacity failure occurs, and $F$ is the factor of safety, which
typically is taken as 3 for the bearing capacity of shallow foundations.

### 3.3.1 Historical Developments

Prandtl (1921) modeled a narrow metal tool bearing against the surface of a block of smooth
softer metal, which later was extended by Reissner (1924) to include a bearing area located
below the surface of the softer metal. The Prandtl-Reissner plastic limit equilibrium plane
strain analysis of a hard object penetrating a softer material later was extended by Terzaghi
(1943) to develop the first rational bearing capacity equation for strip footings embedded in
soils. Terzaghi assumed the soil to be a semi-infinite, isotropic, homogeneous, weightless, rigid
plastic material; the footing to be rigid; and the base of the footing to be sufficiently rough to
ensure there is no separation between the footing and the underlying soil. It also was assumed
that the failure occurs in the general shear mode (Figure 3.7).

### 3.3.2 Terzaghi’s Bearing Capacity Equation

Assuming that the bearing capacity failure occurs in the general shear mode, Terzaghi ex-
pressed his first bearing capacity equation for a strip footing as:
Here, $c$ is the cohesion and $\gamma_1$ and $\gamma_2$ are the unit weights of the soil above and below, respectively, the footing level. $N_c$, $N_q$, and $N_\gamma$ are the bearing capacity factors, which are functions of the friction angle. The ultimate bearing capacity is derived from three distinct components. The first term in Equation 3.10 reflects the contribution of cohesion to the ultimate bearing capacity, and the second term reflects the frictional contribution of the overburden pressure or surcharge. The last term reflects the frictional contribution of the self-weight of the soil in the failure zone.

For square and circular footings, the ultimate bearing capacities are given by Equations 3.11 and 3.12, respectively:

$$q_{ult} = cN_c + \gamma_1 D_f N_q + 0.5B \gamma_2 N_\gamma$$  \hspace{1cm} (3.10)

For square and circular footings, the ultimate bearing capacities are given by Equations 3.11 and 3.12, respectively:

$$q_{ult} = 1.2cN_c + \gamma_1 D_f N_q + 0.4B \gamma_2 N_\gamma$$  \hspace{1cm} (3.11)

$$q_{ult} = 1.2cN_c + \gamma_1 D_f N_q + 0.3B \gamma_2 N_\gamma$$  \hspace{1cm} (3.12)

It must be remembered that the bearing capacity factors in Equations 3.11 and 3.12 are still for strip footings. For local shear failure, where the failure surface is not fully developed and thus the friction and cohesion are not fully mobilized, Terzaghi reduced the values of the friction angle and cohesion by one-third to:

$$\phi' = \tan^{-1}(0.67\phi)$$  \hspace{1cm} (3.13)

$$c' = 0.67c$$  \hspace{1cm} (3.14)

Terzaghi neglected the shear resistance provided by the overburden soil, which was treated as a surcharge (see Figure 3.7). Also, he assumed that $\alpha = \phi$ in Figure 3.7. Subsequent studies by several others show that $\alpha = 45 + \phi/2$ (Vesic 1973), which makes the bearing capacity factors different than what were originally proposed by Terzaghi. With $\alpha = 45 + \phi/2$, the bearing capacity factors $N_q$ and $N_c$ become:

$$N_q = c^\pi \tan \phi \tan^2 \left(45 + \frac{\phi}{2}\right)$$  \hspace{1cm} (3.15)
The above expression for $N_c$ is the same as the one originally proposed by Prandtl (1921), and the expression for $N_q$ is the same as the one given by Reissner (1924). While there is general consensus about Equations 3.15 and 3.16, various expressions for $N_\gamma$ have been proposed in the literature, the most frequently used of which are those proposed by Meyerhof (1963) and Hansen (1970). Some of these different expressions for $N_\gamma$ are presented in Table 3.2.

For undrained loading in clays, when $\phi_u = 0$, it can be shown that $N_q = 1$, $N_\gamma = 0$, and $N_c = 2 + \pi (= 5.14)$. Skempton (1951) studied the variation of $N_c$ with shape and the depth of the foundation. He showed that for a strip footing, it varies from $2 + \pi$ at the surface to 7.5 at a depth greater than 5B, and for a square footing, it varies between $2\pi$ at the surface and 9.0 at a depth greater than 5B. Therefore, for pile foundations, it generally is assumed that $N_c = 9$.

Most of the bearing capacity theories (e.g., Prandtl, Terzaghi) assume that the footing-soil interface is rough. Concrete footings are made by pouring concrete directly on the ground, and therefore the soil-footing interface is rough. Schultze and Horn (1967) noted that from the way concrete footings are cast in place, there is adequate friction at the base, which mobilizes friction angles equal to or greater than $\phi$. Even the bottom of a metal storage tank is not smooth, since the base is always treated with paint or asphalt to resist corrosion (Bowles 1996). Therefore, the assumption of a rough base is more realistic than a smooth one. Based on experimental studies, Vesic (1975) stated that foundation roughness has little effect on the ultimate bearing capacity, provided the footing load is vertical.

Meyerhof’s $N_\gamma$, used predominantly in North America, and Hansen’s, used in Europe, appear to be the most popular of the above. The values of $N_\gamma$ proposed by Meyerhof (1963), Hansen (1970), Vesic (1973), and in Eurocode 7 (European Committee for Standardisation 1995) are shown in Figure 3.8, along with the values of $N_q$ and $N_c$. For $\phi < 30^\circ$, Meyerhof’s and Hansen’s values are essentially the same. For $\phi > 30^\circ$, Meyerhof’s values are larger, the difference increasing with $\phi$. The Indian standard recommends Vesic’s $N_\gamma$ factor (Raj 1995). The Canadian Foundation Engineering Manual recommends Hansen’s $N_\gamma$ factor (Canadian Geotechnical Society 1992).

### 3.3.3 Meyerhof’s Bearing Capacity Equation

In spite of the various improvements that were made to the theoretical developments proposed by Terzaghi, his original form of the bearing capacity equation is still being used because of its
Terzaghi neglected the shear resistance within the overburden soil (i.e., above the footing level), which was included in the modifications made by Meyerhof (1951) that are discussed here. Meyerhof’s (1963) modifications, which are being adapted worldwide, are summarized here. Meyerhof (1963) proposed the general bearing capacity equation of a rectangular footing as

$$q_{ult} = s_c d_i c N_c + s_q d_i q_1 D_i N_q + s_q d_q i_q 0.5 B \gamma_2 N_\gamma$$  \hspace{1cm} (3.17)

where $N_c$, $N_q$, and $N_\gamma$ are the bearing capacity factors of a strip footing. The shape of the footing is accounted for through the shape factors $s_c$, $s_q$, and $s_\gamma$. The depth of the footing is taken into account through the depth factors $d_c$, $d_q$, and $d_\gamma$. The inclination factors $i_c$, $i_q$, and $i_\gamma$ account for the inclination in the applied load. These factors are summarized below.

**Shape factors** (Meyerhof 1963):

$$s_c = 1 + 0.2 \frac{B}{L} \tan^2 \left( 45 + \frac{\phi}{2} \right)$$  \hspace{1cm} (3.18)

$$s_q = s_\gamma = 1 + 0.1 \frac{B}{L} \tan^2 \left( 45 + \frac{\phi}{2} \right) \text{ for } \phi \geq 10^\circ$$  \hspace{1cm} (3.19)

$$s_q = s_\gamma = 1 \text{ for } \phi = 0$$  \hspace{1cm} (3.20)

![FIGURE 3.8 Bearing capacity factors.](image-url)
Depth factors (Meyerhof 1963):

\[
d_c = 1 + 0.2 \frac{D_f}{B} \tan \left(45 + \frac{\phi}{2}\right)
\]

\[
d_q = d_y = 1 + 0.1 \frac{D_f}{B} \tan \left(45 + \frac{\phi}{2}\right) \quad \text{for } \phi \geq 10^\circ
\]

\[
d_q = d_y = 1 \quad \text{for } \phi = 0
\]

Inclination factors (Meyerhof 1963; Hanna and Meyerhof 1981):

\[
i_c = i_q = \left(1 - \frac{\alpha}{90}\right)^2
\]

\[
i_y = \left(1 - \frac{\alpha}{\phi}\right)^2 \quad \text{for } \phi \geq 10^\circ
\]

\[
i_y = 1 \quad \text{for } \phi = 0
\]

In Equations 3.24 and 3.25, \(\alpha\) is the inclination (in degrees) of the footing load to the vertical. It should be noted that in spite of the load being inclined, the ultimate bearing capacity computed from Equation 3.17 gives its vertical component.

3.3.3.1 Plane Strain Correction

It has been reported by several researchers that the friction angle obtained from a plane strain compression test is greater than that obtained from a triaxial compression test by about 4–9° in dense sands and 2–4° in loose sands (Ladd et al. 1977). A conservative estimate of the plane strain friction angle may be obtained from the triaxial friction angle by (Lade and Lee 1976):

\[
\phi_{ps} = 1.5\phi_{tx} - 17^\circ \quad \text{for } \phi_{tx} > 34^\circ
\]

\[
\phi_{ps} = \phi_{tx} \quad \text{for } \phi_{tx} \leq 34^\circ
\]

Allen et al. (2004) related the peak friction angles from direct shear and plane strain compression tests through the following equation:

\[
\phi_{ps} = \tan^{-1}(1.2 \tan \phi_{ds})
\]

The soil element beneath the centerline of a strip footing is subjected to plane strain loading, and therefore, the plane strain friction angle must be used in calculating its bearing capacity. The plane strain friction angle can be obtained from a plane strain compression test. The loading condition of a soil element along the vertical centerline of a square or circular footing more closely resembles axisymmetric loading than plane strain loading, thus requiring
a triaxial friction angle, which can be determined from a consolidated drained or undrained triaxial compression test.

On the basis of the suggestions made by Bishop (1961) and Bjerrum and Kummeneje (1961) that the plane strain friction angle is 10% greater than that from a triaxial compression test, Meyerhof proposed the corrected friction angle for use with rectangular footings as:

$$\phi_{\text{rectangular}} = \left( 1.1 - 0.1 \frac{B}{L} \right) \phi_{tx} \quad (3.30)$$

The above equation simply enables interpolation between $\phi_{tx}$ (for $B/L = 1$) and $\phi_{ps}$ (for $B/L = 0$). The friction angles available in most geotechnical designs are derived from triaxial tests in the laboratory or *in situ* penetration tests.

### 3.3.3.2 Eccentric Loading

When the footing is applied with some eccentricity, the ultimate bearing capacity is reduced. Meyerhof (1963) suggested the effective footing breadth ($B'$) and length ($L'$) as:

$$B' = B - 2e_B \quad (3.31)$$

$$L' = L - 2e_L \quad (3.32)$$

where $e_B$ and $e_L$ are the eccentricities along the breadth and length, respectively, as shown in Figure 3.9.

For footings with eccentricities, $B'$ and $L'$ should be used in computing the ultimate bearing capacity (Equation 3.17) and shape factors (Equations 3.18 and 3.19). In computing the depth factors (Equations 3.21 and 3.22), $B$ should be used. The unhatched area ($A' = B' \times L'$) in Figure 3.9 is the effective area which contributes to the bearing capacity, and therefore, the ultimate footing load is computed by multiplying the ultimate bearing capacity by this area $A'$. It should be noted that when the hatched area is disregarded, the load is applied at the center of the remaining area.

### 3.3.4 Hansen’s Bearing Capacity Equation

Based on theoretical and experimental work, Hansen (1970) and Vesic (1973, 1975) proposed the following bearing capacity equation for drained and undrained conditions:

$$q_{ult} = s_c d_c i_c b_c g_c cN_c + s_q d_q i_q b_q g_q \gamma D_f N_q + s_\gamma d_\gamma i_\gamma b_\gamma g_\gamma 0.5B\gamma N_\gamma \quad (3.33)$$
In addition to the shape \( s \), depth \( d \), and inclination \( i \) factors, they included base inclination \( b \) and ground inclination \( g \) factors. Base inclination factors account for any inclination in the base of the footing. This may become necessary when the footing is required to carry an inclined load. The ground inclination factors account for the reduction in bearing capacity when the footing is located on sloping ground, as shown in Figure 3.10. The equations to compute these factors are summarized below.

**Shape factors** (Hansen 1970):

\[
s_c = 1 + \left( \frac{B}{L} \right) \left( \frac{N_q}{N_c} \right)
\]

\[
s_q = 1 + \left( \frac{B}{L} \right) \tan \phi
\]

\[
s_g = 1 - 0.4 \left( \frac{B}{L} \right)
\]

**Depth factors** (Hansen 1970):

\[
d_c = 1 + 0.4 \frac{D_f}{B}
\]

\[
d_q = 1 + 2 \frac{D_f}{B} \tan \phi (1 - \sin \phi)^2
\]

\[
d_g = 1
\]

When \( D_f > B \), the factor \( D_f/B \) should be replaced by \( \tan^{-1} (D_f/B) \).
Load inclination factors (Hansen 1970):

\[ i_c = 0.5 + 0.5 \left( \frac{H}{cBL} \right) \]  
for \( \phi = 0 \) \hspace{1cm} (3.40)

\[ i_c = i_q - \frac{1 - i_q}{N_q - 1} \]  
for \( \phi > 0 \) \hspace{1cm} (3.41)

\[ i_q = \left( 1 - \frac{0.5H}{V + cBL \cot \phi} \right)^5 \] \hspace{1cm} (3.42)

\[ i_\gamma = \left[ 1 - \left( \frac{0.7 - \frac{\theta^\circ}{450}}{V + cBL \cot \phi} \right)H \right]^5 \] \hspace{1cm} (3.43)

The cohesion mobilized at the footing-soil contact area must be used for \( c \) in Equations 3.40, 3.42, and 3.43. The U.S. Army (1993) recommends using adhesion or a reduced value of cohesion.

Base inclination factors (Hansen 1970):

\[ b_c = 1 - \frac{\theta^\circ}{147} \] \hspace{1cm} (3.44)

\[ b_q = \exp(-0.0349\theta^\circ \tan \phi) \] \hspace{1cm} (3.45)

\[ b_\gamma = \exp(-0.0471\theta^\circ \tan \phi) \] \hspace{1cm} (3.46)

Ground inclination factors (Hansen 1970):

\[ g_c = 1 - \frac{\beta^\circ}{147} \] \hspace{1cm} (3.47)

\[ g_q = g_\gamma = (1 - 0.5 \tan \beta)^5 \] \hspace{1cm} (3.48)

3.3.5 Vesic’s Bearing Capacity Equation

Vesic’s bearing capacity equation is the same as Hansen’s, but with slight differences in the bearing capacity factor \( N_\gamma \) and the last three inclination factors \( i, b, \) and \( g \), which are less conservative.
Shape factors (Vesic 1975):

\[ s_c = 1 + \left( \frac{B}{L} \right) \left( \frac{N_q}{N_c} \right) \]  
(3.49)

\[ s_q = 1 + \left( \frac{B}{L} \right) \tan \phi \]  
(3.50)

\[ s_\gamma = 1 - 0.4 \left( \frac{B}{L} \right) \]  
(3.51)

Depth factors (Vesic 1975):

\[ d_c = 1 + 0.4 \frac{D_f}{B} \quad \text{for } \phi = 0 \]  
(3.52)

\[ d_c = d_q - \frac{1 - d_q}{N_q - 1} \quad \text{for } \phi > 0 \]  
(3.53)

\[ d_q = 1 + 2 \frac{D_f}{B} \tan \phi (1 - \sin \phi)^2 \]  
(3.54)

\[ d_\gamma = 1 \]  
(3.55)

When \( D_f > B \), the factor \( D_f/B \) should be replaced by \( \tan^{-1} (D_f/B) \).

Load inclination factors (Vesic 1975):

If \( V \) and \( H \) are the components of the load perpendicular and parallel to the base of the footing, the load inclination factors \( i_c \), \( i_q \), and \( i_\gamma \) are given by:

\[ i_c = 1 - \frac{mH}{AcN_c} \quad \text{for } \phi = 0 \]  
(3.56)

\[ i_c = i_q - \frac{1 - i_q}{N_q - 1} \quad \text{for } \phi > 0 \]  
(3.57)

\[ i_q = \left( 1 - \frac{H}{V + cBL \cot \phi} \right)^m \]  
(3.58)

\[ i_\gamma = \left( 1 - \frac{H}{V + cBL \cot \phi} \right)^{m+1} \]  
(3.59)
where

\[ m = \frac{2 + B/L}{1 + B/L} \]

if the load is inclined in the direction parallel to the breadth and

\[ m = \frac{2 + L/B}{1 + L/B} \]

if the load is inclined in the direction parallel to the length. The cohesion mobilized at the footing-soil contact area must be used for \( c \) in Equations 3.56, 3.58, and 3.59. The U.S. Army (1993) recommends using adhesion or a reduced value of cohesion.

**Base inclination factors** (Vesic 1975):

\[ b_c = 1 - \frac{\phi^\circ}{147} \quad \text{for } \phi = 0 \]  \hspace{1cm} (3.60)

\[ b_c = b_q - \frac{1 - b_q}{N_q - 1} \quad \text{for } \phi > 0 \]  \hspace{1cm} (3.61)

\[ b_q = b_q' = \left(1 - \frac{\theta^\circ \tan \phi}{57}\right)^2 \]  \hspace{1cm} (3.62)

where \( \theta \) is the inclination (in degrees) of the base of the footing to horizontal (see Figure 3.10).

**Ground inclination factors** (Vesic 1975):

\[ g_c = 1 - \frac{\beta^\circ}{147} \quad \text{for } \phi = 0 \]  \hspace{1cm} (3.63)

\[ g_c = g_q - \frac{1 - g_q}{N_q - 1} \quad \text{for } \phi > 0 \]  \hspace{1cm} (3.64)

\[ g_q = g_q' = (1 - \tan \beta)^2 \]  \hspace{1cm} (3.65)

where \( \beta \) is the inclination of the slope in degrees, \( \beta < \phi \), and \( \theta + \beta \leq 90^\circ \) (see Figure 3.10). On a sloping ground, when \( \phi = 0 \), \( N_q = -2 \sin \beta \).

It should be noted that the ultimate bearing capacity equation for clays under undrained conditions (\( \phi_u = 0 \)) sometimes is given in the literature slightly differently as (Aysen 2002; Bowles 1988)

\[ q_{\text{ult}} = (1 + s_c + d_c - i_c - b_c - g_c) c_u N_c + \gamma D_f \]  \hspace{1cm} (3.66)
and consequently the reported correction factors for Equation 3.32 are slightly different (U.S. Army 1993; Cernica 1995; Coduto 2001; McCarthy 2007; European Committee for Standardisation 1995).

### 3.3.6 Gross and Net Pressures and Bearing Capacities

The ultimate bearing capacities computed using Equations 3.10–3.12, 3.17, 3.33, and 3.66 are all gross ultimate bearing capacities. There already is an overburden pressure of $\gamma D_f$ acting at the foundation level. The net ultimate bearing capacity is the maximum additional soil pressure that can be sustained before failure. Therefore, net ultimate bearing capacity is obtained by subtracting the overburden pressure from the gross ultimate bearing capacity. Similarly, the net applied pressure is the additional pressure applied at the foundation level in excess of the existing overburden pressure. The safety factor with respect to bearing capacity failure is therefore defined in terms of the net values as:

$$F = \frac{q_{\text{ult,net}}}{q_{\text{applied,net}}} = \frac{q_{\text{ult,gross}} - \gamma D_f}{q_{\text{applied,gross}} - \gamma D_f}$$  \hspace{1cm} (3.67)

In most spread footing designs, the gross pressures are significantly larger than the overburden pressures. Only in problems that involve removal of large overburden pressures, such as foundations for basements, can gross and net pressures be significantly different. In clays under undrained conditions ($\phi_u = 0$), $N_c = 5.14$, $N_q = 1$, and $N_{\gamma} = 0$. Therefore, the net ultimate bearing capacity of a shallow foundation can be written as:

$$q_{\text{ult,net}} = 5.14 c_u \left( 1 + 0.2 \frac{D_f}{B} \right) \left( 1 + 0.2 \frac{B}{L} \right)$$  \hspace{1cm} (3.68)

### 3.3.7 Effects of the Water Table

When computing the ultimate bearing capacity in terms of effective stress parameters, it is necessary to use the correct unit weights, depending on the location of the water table. If the water table lies at or above ground level, $\gamma'$ must be used in both terms in the bearing capacity equation (Equation 3.10). If the water table lies at the footing level, $\gamma_m$ must be used in the second term and $\gamma'$ in the third term in the bearing capacity equation. It can be seen from Figure 3.7 that the failure zone within the soil is confined to a depth of $B$ below the footing width. Therefore, if the water table lies at $B$ or more below the footing, the bulk unit weight ($\gamma_m$) must be used in both terms in the bearing capacity equation. Terzaghi and Peck (1967) stated that the friction angle is reduced by 1–2° when a sand is saturated. Therefore, if a future rise in the water table is expected, the friction angle may be reduced slightly in computing the ultimate bearing capacity.

### 3.3.8 Presumptive Bearing Pressures

Presumptive bearing pressures are very approximate and conservative safe bearing pressures that can be assumed in preliminary designs. They are given in building codes and geotechnical textbooks (see U.S. Army 1993; Bowles 1988). Here, the specified values do not reflect the site
or geologic conditions, shear strength parameters, or the foundation dimensions. Some typical values are given in Table 3.3.

### 3.4 Pressure Distribution beneath Eccentrically Loaded Footings

The pressure distribution beneath a flexible footing often is assumed to be uniform if the load is applied at the center. This is not the case when the load is applied with some eccentricity in one or both directions. Eccentricity can be introduced through moments and/or lateral loads such as wind loads. It can reduce the ultimate bearing capacity, and with the reduced effective area, the allowable load on the footing is reduced even further.

In a strip footing, when the line load is applied with an eccentricity of \( e \), as shown in Figure 3.11a, the soil pressure at any point beneath the footing is given by

\[
q(x) = \frac{Q}{B} \left( 1 + \frac{12ex}{B^2} \right)
\]  

(3.69)

where \( x \) is the horizontal distance from the centerline. The maximum and minimum values of the soil pressure, which occur at the two edges of the strip footing, at \( x = 0.5B \) and \( x = -0.5B \), respectively, are given by:

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Bearing Capacity (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rocks</strong></td>
<td></td>
</tr>
<tr>
<td>Hard and sound igneous and gneissic rock</td>
<td>10,000</td>
</tr>
<tr>
<td>Hard limestone/sandstone</td>
<td>4,000</td>
</tr>
<tr>
<td>Schist/slate</td>
<td>3,000</td>
</tr>
<tr>
<td>Hard shale/mudstone or soft sandstone</td>
<td>2,000</td>
</tr>
<tr>
<td>Soft shale/mudstone</td>
<td>600–1,000</td>
</tr>
<tr>
<td>Hard sound chalk or soft limestone</td>
<td>600</td>
</tr>
<tr>
<td><strong>Granular soils</strong></td>
<td></td>
</tr>
<tr>
<td>Dense gravel or sand/gravel</td>
<td>&gt;600</td>
</tr>
<tr>
<td>Medium-dense gravel or sand/gravel</td>
<td>200–600</td>
</tr>
<tr>
<td>Loose gravel or sand/gravel</td>
<td>&lt;200</td>
</tr>
<tr>
<td>Dense sand</td>
<td>&gt;300</td>
</tr>
<tr>
<td>Medium-dense sand</td>
<td>100–300</td>
</tr>
<tr>
<td>Loose sand</td>
<td>&lt;100</td>
</tr>
<tr>
<td><strong>Cohesive soils</strong></td>
<td></td>
</tr>
<tr>
<td>Very stiff clays</td>
<td>300–600</td>
</tr>
<tr>
<td>Stiff clays</td>
<td>150–300</td>
</tr>
<tr>
<td>Firm clays</td>
<td>75–150</td>
</tr>
<tr>
<td>Soft clays and silts</td>
<td>&lt;75</td>
</tr>
</tbody>
</table>

It can be seen from Equation 3.71 that the soil pressure beneath the footing will be compressive at all points provided $e < B/6$. Since there cannot be tensile normal stress between the foundation and the soil, when $e$ exceeds $B/6$, one edge of the footing will lift off the ground, reducing the contact area, resulting in redistribution of the contact pressure. It is therefore desirable to limit the eccentricity to a maximum of $B/6$.

In a rectangular footing with eccentricities of $e_B$ and $e_L$ in the direction of breadth and length, respectively, the contact pressure at any point beneath the footing is given by:

$$q(x, y) = \frac{Q}{BL} \left( 1 + \frac{12e_B}{B^2} x + \frac{12e_L}{L^2} y \right)$$

(3.72)

Here, the origin is at the center of the footing and the $x$- and $y$-axes are in the direction of breadth and length, respectively (see Figure 3.11b). The lightly shaded area at the center of Figure 3.11b, a rhombus, is known as the kern. Provided the foundation load acts within this area, the contact stresses are compressive at all points beneath the footing.

### 3.5 Settlement of Shallow Foundations in Cohesive Soils

When foundations are subjected to vertical loads, there will be settlement. Depending on whether the underlying soils are cohesive or granular, the settlement pattern can be quite
different. In saturated cohesive soils, the settlements consist of three components: immediate settlement \( s_i \), consolidation settlement \( s_c \), and secondary compression \( s_s \). Immediate settlement occurs immediately after the load is applied and is instantaneous. Consolidation settlement occurs due to the expulsion of water from the soil and dissipation of excess pore water pressure. This can take place over a period of several years. Secondary compression settlement, also known as creep, occurs after the consolidation is completed. Therefore, there will be no excess pore water pressure during the secondary compression stage.

### 3.5.1 Immediate Settlement

Immediate settlement, also known as distortion settlement, initial settlement, or elastic settlement, occurs immediately upon the application of the load, due to lateral distortion of the soil beneath the footing. In clays, where drainage is poor, it is reasonable to assume that immediate settlement takes place under undrained conditions where there is no volume change (i.e., \( v = 0.5 \)). The average immediate settlement under a flexible footing generally is estimated using the theory of elasticity, using the following equation, originally proposed by Janbu et al. (1956):

\[
s_i = \frac{qB}{E_u} \mu_0 \mu_1
\]

(3.73)

The values of \( \mu_1 \) and \( \mu_2 \), originally suggested by Janbu et al. (1956), were modified later by Christian and Carrier (1978), based on the work by Burland (1970) and Giroud (1972). The values of \( \mu_0 \) and \( \mu_1 \), assuming \( v = 0.5 \), are given in Figure 3.12. Obtaining a reliable estimate of the undrained Young’s modulus \( (E_u) \) of clays through laboratory or in situ tests is quite difficult. It can be estimated using Figure 3.13, proposed by Duncan and Buchignani (1976) and the U.S. Army (1994). \( E_u/\epsilon_u \) can vary from 100 for very soft clays to 1500 for very stiff clays.

Typical values of the elastic modulus for different types of clays are given in Table 3.4. Immediate settlement generally is a small fraction of the total settlement, and therefore a rough estimate often is adequate.

### 3.5.2 Consolidation Settlement

Consolidation is a time-dependent process in saturated clays, where the foundation load is gradually transferred from the pore water to the soil skeleton. Immediately after loading, the entire applied normal stress is carried by the water in the voids, in the form of excess pore water pressure. With time, the pore water drains out into the more porous granular soils at the boundaries, thus dissipating the excess pore water pressure and increasing the effective stresses. Depending on the thickness of the clay layer, and its consolidation characteristics, this process can take from a few days to several years.

Consolidation settlement generally is computed assuming one-dimensional consolidation, and then a correction factor is applied for three-dimensional effects (Skempton and Bjerrum 1957). In one-dimensional consolidation, the normal strains and drainage are assumed to take place only in the vertical direction. This situation arises when the applied pressure at the ground level is uniform and is of a very large lateral extent, as shown in Figure 3.14.

<table>
<thead>
<tr>
<th>Clay</th>
<th>( E ) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very soft clay</td>
<td>0.5–5</td>
</tr>
<tr>
<td>Soft clay</td>
<td>5–20</td>
</tr>
<tr>
<td>Medium clay</td>
<td>20–50</td>
</tr>
<tr>
<td>Stiff clay, silty clay</td>
<td>50–100</td>
</tr>
<tr>
<td>Sandy clay</td>
<td>25–200</td>
</tr>
<tr>
<td>Clay shale</td>
<td>100–200</td>
</tr>
</tbody>
</table>

In a clay layer with an initial thickness of $H$ and a void ratio of $e_0$, the final consolidation settlement $s_c$ due to the applied pressure $q$ can be estimated from

$$s_c = \frac{\Delta e}{1 + e_0} H$$

(3.74)

where $\Delta e$ is the change in the void ratio due to the applied pressure $q$. $H$ and $e_0$ can be obtained from the soil data, and $\Delta e$ has to be computed as follows.
Three different cases, as shown in Figure 3.15, are discussed here. Point I corresponds to the initial state of the clay, where the void ratio and the vertical stress are $e_0$ and $\sigma'_{vo}$, respectively. With the vertical stress increase of $\Delta \sigma_v$, consolidation takes place, and the void ratio decreases by $\Delta e$. Point F corresponds to the final state, at the end of consolidation. Point P corresponds to the preconsolidation pressure ($\sigma'_p$) on the virgin consolidation line.

**Case I.** If the clay is normally consolidated, $\Delta e$ can be computed from:

$$
\Delta e = C_c \log \left( \frac{\sigma'_{vo} + \Delta \sigma_v}{\sigma'_{vo}} \right)
$$

(3.75)
Case II. If the clay is overconsolidated and $\sigma'_vo + \Delta \sigma_v \leq \sigma'_p$ (i.e., the clay remains overconsolidated at the end of consolidation), $\Delta e$ can be computed from:

$$\Delta e = C_r \log \left( \frac{\sigma'_vo + \Delta \sigma_v}{\sigma'_vo} \right)$$

(3.76)

Case III. If the clay is overconsolidated and $\sigma'_vo + \Delta \sigma_v \geq \sigma'_p$ (i.e., the clay becomes normally consolidated at the end of consolidation), $\Delta e$ can be computed from:

$$\Delta e = C_r \log \left( \frac{\sigma'_p}{\sigma'_vo} \right) + C_c \log \left( \frac{\sigma'_vo + \Delta \sigma_v}{\sigma'_p} \right)$$

(3.77)

In one-dimensional consolidation, assuming the pressure at the ground level is applied over a large lateral extent, $\Delta \sigma_v = q$ at any depth. In the case of footings where the loading is not one-dimensional, $\Delta \sigma_v$ can be significantly less than the footing pressure $q$ and can be estimated using the methods discussed in Section 3.2.

Another but less desirable method to compute the consolidation settlement is to use the coefficient of volume compressibility ($m_v$). The final consolidation settlement can be written as:

$$s_c = m_v q H$$

(3.78)

The main problem with this apparently simple method is that $m_v$ is stress dependent, and therefore a value appropriate to the stress level must be used. The consolidation settlement $s(t_1)$ at a specific time $t_1$ can be determined from the $U_{avg}-T$ plot in Figure 1.17.

3.5.3 Secondary Compression Settlement

Secondary compression settlement takes place at constant effective stress, when there is no more dissipation of excess pore water pressure. For simplicity, it is assumed to start occurring...
when the primary consolidation is completed at time $t_p$ (see Figure 3.16), and the settlement increases linearly with the logarithm of time. Secondary compression settlement can be estimated using the following equation:

$$s_s = C_\alpha \left( \frac{H}{1 + e_p} \right) \log \left( \frac{t}{t_p} \right) \quad \text{for } t > t_p$$

(3.79)

Here, $e_p$ is the void ratio at the end of primary consolidation and $C_\alpha$ is the coefficient of secondary compression or the secondary compression index, which can be determined from a consolidation test or estimated empirically. Assuming that the void ratio decreases linearly with the logarithm of time, $C_\alpha$ is defined as:

$$C_\alpha = \frac{\Delta e}{\Delta \log t}$$

(3.80)

Mesri and Godlewski (1977) reported that $C_\alpha / C_c$ is a constant for a specific soil and suggested typical values. In the absence of consolidation test data, $C_\alpha$ can be assumed to be 0.03–0.08 times $C_c$. While the upper end of the range applies to organic and highly plastic clays, the lower end of the range is suitable for inorganic clays. Secondary compression settlement can be quite significant in organic clays, especially in peat.

### 3.6 Settlement of Shallow Foundations in Granular Soils

Settlement of footings in granular soils is instantaneous, with some possibility for long-term creep. There are more than 40 different settlement prediction methods, but the quality of the predictions is still very poor, as demonstrated at the Settlement 94 settlement prediction symposium in Texas in 1994 (Briaud and Gibbens 1994).

The five most important factors that govern the settlement of a footing are the applied pressure, soil stiffness, footing breadth, footing depth, and footing shape. Soil stiffness often
is quantified indirectly through penetration resistance such as the $N$-value or blow count from a standard penetration test or through tip resistance $q_c$ from a cone penetration test. Das and Sivakugan (2007) summarized the empirical correlations relating soil stiffness to penetration resistance.

### 3.6.1 Terzaghi and Peck Method

Terzaghi and Peck (1967) proposed the first rational method for predicting settlement of a shallow foundation in granular soils. They related the settlement of a square footing of width $B$ (in meters) to that of a 300-mm square plate, obtained from a plate loading test, through the following expression:

$$\delta_{\text{footing}} = \delta_{\text{plate}} \left( \frac{2B}{B + 0.3} \right)^2 \left( 1 - \frac{1}{4} \frac{D_f}{B} \right)$$  \hspace{1cm} (3.81)

The last term in Equation 3.81 accounts for the reduction in settlement with the increase in footing depth. Leonards (1986) suggested replacing $\frac{1}{4}$ by $\frac{1}{3}$, based on additional load test data. The values of $\delta_{\text{plate}}$ can be obtained from Figure 3.17, which summarizes the plate loading test data given by Terzaghi and Peck (1967). This method originally was proposed for square footings, but can be applied to rectangular and strip footings with caution. The deeper influence zone and increase in the stresses within the soil mass in the case of rectangular or strip footings are compensated for by the increase in the soil stiffness.

![Figure 3.17 Settlement of 300-mm x 300-mm plate (adapted from Terzaghi et al. 1996; load test data from late Professor G.A. Leonards).](image)
3.6.2 Schmertmann et al. Method

Based on the theory of elasticity, Schmertmann (1970) proposed that the vertical normal strain ($\varepsilon_z$) at a depth $z$ below the footing is given by

$$\varepsilon_z = \frac{q}{E_z I_z}$$

(3.82)

where $E_z$ and $I_z$ are Young's modulus and the strain influence factor, respectively, at depth $z$. Based on some finite element studies and load tests on model footings, Schmertmann proposed the influence factor as shown in Figure 3.18a, which is known as the $2B$-0.6 distribution. The influence factor increases linearly from 0 at the footing level to 0.6 at a depth of $0.5B$ below the footing and then decreases linearly to 0 at a depth of $2B$ below the footing. Integrating the above equation and dividing the granular soil beneath the footing into sublayers of constant Young's modulus, the vertical settlement can be expressed as

$$s = q_{net} C_1 C_2 \sum_{z=0}^{z=2B} \frac{I_z dz}{E_z}$$

(3.83)

where $C_1$ and $C_2$ are the correction factors to account for the embedment and strain relief due to the removal of overburden and the time dependence of settlement, respectively, and $q_{net}$ is the net applied pressure at the footing level. $C_1$ and $C_2$ are given by

$$C_1 = 1 - 0.5 \left( \frac{\sigma_{vo}'}{q_{net}} \right) \geq 0.5$$

(3.84)

FIGURE 3.18 Schmertmann et al.’s influence factors.
Design of Shallow Foundations

\[ C_2 = 1 + 0.2 \log \left( \frac{t}{0.1} \right) \]  

(3.85)

where \( \sigma'_{vo} \) is the effective in situ overburden stress at the footing level, and \( t \) is the time since loading (in years). Leonards (1986), Holtz (1991), and Terzaghi et al. (1996) suggest that \( C_2 = 1 \), disregarding the time-dependent settlements in granular soils. They suggest that the time-dependent settlements in the footings studied by Schmertmann probably are due to the thin layers of clays and silts interbedded within the sands in Florida, from where most of Schmertmann’s load test data come. Schmertmann (1970) recommended that Young’s modulus be derived from the static cone resistance as \( E = 2q_c \). Leonards (1986) suggested that \( E \) (kg/cm²) = 8\( N_{60} \) for normally consolidated sands, where \( N_{60} \) is the blow count from a standard penetration test, not corrected for overburden (1 kg/cm² = 98.1 kPa).

Schmertmann’s (1970) original method does not take the footing shape into account. Realizing the need to account for the footing shape, Schmertmann et al. (1978) made some modifications to the original method. The modified influence factor diagram is shown in Figure 3.18b, where the strain influence factor extends to a depth of 2\( B \) for square footings and 4\( B \) for strip footings, peaking at depths of 0.5\( B \) and \( B \), respectively. The peak value of the influence factor is given by

\[ I_{z,\text{peak}} = 0.5 + 0.1 \sqrt{\frac{q_{\text{net}}}{\sigma'_{vo}}} \]  

(3.86)

where \( \sigma'_{vo} \) is the original overburden pressure at a depth of 0.5\( B \) below the footing for square footings and \( B \) below the footing for strip footings, where the peak values occur. The equations for computing the settlement and the correction factors remain the same. Schmertmann et al. (1978) suggested that \( E = 2.5q_c \) for axisymmetric loading and \( E = 3.5q_c \) for plane strain loading, based on the observation by Lee (1970) that Young’s modulus is about 40% greater for plane strain loading compared to axisymmetric loading. They suggested that for rectangular footings, the settlement be calculated separately for \( B/L = 0 \) and 1 and interpolated on the basis of \( B/L \).

Terzaghi et al. (1996) suggested a simpler influence factor diagram, shown in Figure 3.18c, with the influence factors starting and peaking at the same points but extending to depths of 2\( B \) and 4\( B \) for square and strip footings. For rectangular footings, they suggested an interpolation function to estimate the depth of influence \( z_I \) (see Figure 3.18c) as:

\[ z_I = 2B \left( 1 + \log \frac{L}{B} \right) \]  

(3.87)

Terzaghi et al. (1996) suggest taking \( E = 3.5q_c \) for axisymmetric loading and increasing it by 40% for plane strain loading and suggest the following expression for \( E \) of a rectangular footing:

\[ E_{\text{rectangular footing}} = 3.5 \left( 1 + 0.4 \log \frac{L}{B} \right) q_c \]  

(3.88)
These modifications give more realistic and less conservative estimates of settlements. Nevertheless, the above values of \( E = (3.5–4.9)q_c \) are significantly larger than what is recommended in the literature.

### 3.6.3 Burland and Burbidge Method

Burland et al. (1977) collated more than 200 settlement records of shallow foundations of buildings, tanks, and embankments on granular soils and plotted settlement per unit pressure against the footing breadth, as shown in Figure 3.19, defining the upper limits for possible settlement that can be expected. It is a good practice to use this figure to check whether the settlement predicted by a specific method falls within the bounds. They suggested that the “probable” settlement is about 50% of the upper limit shown in the figure and that in most cases the maximum settlement will be unlikely to exceed 75% of the upper limit.

Burland and Burbidge (1985) reviewed the above settlement records and proposed an indirect and empirical method for estimating settlement of shallow foundations in granular soils, based on \( N \)-values from standard penetration tests that are not corrected for overburden pressure. The influence depth \( z_I \) was defined as

\[
    z_I = B^{0.7}
\]  

(3.89)

where \( z_I \) and \( B \) are in meters. They expressed the compressibility of the soil by a compressibility index \( (I_c) \), which is similar to the coefficient of volume compressibility \( (m_v) \) used in consolidation of saturated clays. For normally consolidated granular soils, \( I_c \) was related to the average blow count within the influence depth \( N_{60} \) by

---

**FIGURE 3.19** Upper limits of settlement per unit pressure (after Burland et al. 1977).
\[ I_c = \frac{1.71}{N_{60}^{1.4}} \] (3.90)

where \( I_c \) is in MPa\(^{-1}\). For overconsolidated granular soils, \( I_c \) is one-third of what is given in Equation 3.90. Burland and Burbidge (1985) suggested that the settlement can be estimated from:

\[ s = qI_c z_I \] (3.91)

It should be noted that Equation 3.91 is similar in form to Equation 1.36, which is used for estimating consolidation settlement in clays. In normally consolidated granular soils, Equation 3.91 becomes:

\[ s = q \frac{1.71}{N_{60}^{1.4}} B^{0.7} \] (3.92)

In overconsolidated granular soils, if the preconsolidation pressure (\( \sigma'_p \)) can be estimated, Equation 3.91 becomes:

\[ s = \frac{1}{3} q \frac{1.71}{N_{60}^{1.4}} B^{0.7} \text{ if } q \leq \sigma'_p \] (3.93)

\[ s = \left( q - \frac{2}{3} \sigma'_p \right) \frac{1.71}{N_{60}^{1.4}} B^{0.7} \text{ if } q \geq \sigma'_p \] (3.94)

For fine sands and silty sands below the water table, where \( N_{60} > 15 \), driving of the split-spoon sampler can dilate the sands, which can produce negative pore water pressure that would increase the effective stresses and hence overestimate the blow count. Here, Terzaghi’s correction given below should be applied:

\[ N_{60, \text{corrected}} = 15 + 0.5(N_{60} - 15) \] (3.95)

In gravel or sandy gravel, \( N \) should be increased by 25% using Equation 3.96:

\[ N_{60, \text{corrected}} = 1.25N_{60} \] (3.96)

The settlements estimated above apply to square footings. For rectangular or strip footings, settlement has to be multiplied by the following factor (\( f_s \)):

\[ f_s = \left( \frac{1.25L/B}{0.25 + L/B} \right)^2 \] (3.97)

The settlement estimated above implies that there is granular soil to a depth of at least \( z_I \). If the thickness (\( H_s \)) of the granular layer below the footing is less than the influence depth, the settlement has to be multiplied by the following reduction factor (\( f_l \)): 
Burland and Burbidge (1985) noted some time-dependent settlement of footings and suggested a multiplication factor \( f_t \) given by

\[
    f_t = 1 + R_3 + R_t \log \left( \frac{t}{3} \right)
\]

where \( R_3 \) takes into consideration the time-dependent settlement during the first three years of loading, and the last component accounts for the time-dependent settlement that takes place after the first three years at a slower rate. Suggested values for \( R_3 \) and \( R_t \) are 0.3–0.7 and 0.2–0.8, respectively. The lower end of the range is applicable for static loads and the upper end for fluctuating loads such as bridges, silos, and tall chimneys.

### 3.6.4 Accuracy and Reliability of the Settlement Estimates and Allowable Pressures

Das and Sivakugan (2007) reviewed the different settlement prediction methods and discussed the current state-of-the-art. The three methods discussed above in detail are the most popular for estimating settlement of shallow foundations in granular soils. It is well known that these methods overestimate settlement in general and thus are conservative. Sivakugan et al. (1998) studied 79 settlement records where the footing width was less than 6 m and concluded that the settlement predictions by Terzaghi and Peck (1967) and Schmertmann (1970) overestimate settlement by about 2.18 and 3.39 times, respectively.

Tan and Duncan (1991) introduced two parameters—accuracy and reliability—to quantify the quality of settlement predictions and applied these to 12 different methods using a large database of settlement records. *Accuracy* was defined as the average ratio of the predicted settlement to the measured settlement. *Reliability* is the probability that the predicted settlement is greater than the measured settlement. Therefore, an ideal settlement prediction method will have an accuracy of 1 and reliability approaching 100%. There often is a trade-off between accuracy and reliability. The Terzaghi and Peck (1967) method has high reliability but poor accuracy, which shows that the estimates are conservative. On the other hand, the Burland and Burbidge (1985) method has good accuracy but poor reliability, which shows that the predictions are more realistic, but it does not always overestimate like the Schmertmann et al. (1978) method or Terzaghi and Peck (1967) method and is less conservative.

It is widely documented in the literature that the design of shallow foundations in granular soils is almost always governed more by settlement considerations than bearing capacity. Therefore, more care is required in the settlement computations. The allowable bearing capacity values, on the basis of limiting settlement to 25 mm, estimated by the Burland and Burbidge (1985) and Terzaghi and Peck (1967) methods are shown in Figure 3.20. The Burland and Burbidge (1985) charts were developed for square footings on normally consolidated sands, with no consideration given to time-dependent settlement. If the sand is overconsolidated with a preconsolidation pressure of \( \sigma_p' \) and \( q < \sigma_p' \), the allowable pressure from Figure 3.20 should be multiplied by 3. If the sand is overconsolidated and \( q > \sigma_p' \), add 0.67\( \sigma_p' \) to the value obtained from Figure 3.20. For any other value of limiting settlement, the
allowable pressure from Figure 3.20 must be adjusted proportionally. To limit the probability that the actual settlement will exceed 25 mm, it may be necessary to limit the maximum settlement to 16 mm and reduce the allowable soil pressure proportionately (Terzaghi et al. 1996). It can be seen in Figure 3.20 that the Burland and Burbidge (1985) method gives significantly smaller settlements and higher allowable pressures compared to the more conservative Terzaghi and Peck (1967) method.

Meyerhof (1956, 1974) suggested an expression for allowable pressure that would limit settlement to 25 mm, which again underestimates the allowable pressure significantly. Bowles (1996) suggested increasing this value by 50%, whereby the modified Meyerhof equation becomes

$$q_{\text{allowable}} = 1.25N_{60} \left( \frac{B + 0.3}{B} \right)^2 \left( 1 + \frac{1}{3} \frac{D_f}{B} \right)$$

(3.100)

where $D_f/B \leq 1$ and $B > 1.2$ m. Equation 3.100 gives a slightly higher allowable pressure than the Terzaghi and Peck (1967) values, but significantly less than the Burland and Burbidge (1985) values shown in Figure 3.20.

### 3.6.5 Probabilistic Approach

The magnitude of settlement can have a different meaning depending on which method was used in the settlement computations. Sivakugan and Johnson (2004) proposed a probabilistic design chart, based on several settlement records reported in the literature, to quantify the
probability that the settlement predicted by a certain method will exceed a specific limiting value in the field. Three separate charts for the Terzaghi and Peck, Schmertmann et al., and Burland and Burbidge methods are given in Figure 3.21. For example, if the settlement predicted by the Schmertmann et al. method is 20 mm, the probability that the actual settlement will exceed 25 mm is 0.2.

3.7 Raft Foundations

A raft foundation, also known as a mat foundation, is a large, thick concrete slab that supports all or some of the columns and/or walls of a structure. A raft also can support an entire structure, such as a silo, storage tank, chimney, tower, and foundation machinery. A hollow raft can reduce the heavy self-weight of a large slab and yet provide enough structural stiffness. A widely accepted practical criterion is to use a raft when more than 50% of the building plan projection is covered by footings. Other purposes of rafts include increasing the foundation area and thus increasing the foundation bearing capacity whenever possible, bridging over small compressible pockets to minimize differential settlements, resisting hydrostatic uplift, facilitating basement waterproofing, and redistributing horizontal soil and water thrust through the structure or support peripheral columns and walls. Reduced raft thicknesses can be achieved efficiently by introducing structural stiffeners such as plates thickened under columns, a two-way beam and slab, and a flat plate with pedestals or basement walls incorporated into the raft (Teng 1975; Bowles 1996). Thinner slabs also can be designed to resist high uplift pressures by introducing vertical prestressed anchors or tension piles (Danziger et al. 2006). Compared to footings, a raft spreads the structural load over a larger area in the soil and reduces the bearing pressure. Because of the high stiffness of the thick concrete slab, a raft can reduce differential settlement. Differential settlement also can be minimized by simultaneously taking into account the slab stiffness and the stiffness of the superstructure.

3.7.1 Structural Design Methods for Rafts

The methods for raft foundation design are classified as rigid and flexible. The rigid method (also known as the conventional method) is still widely used in practice because of its simplicity. It is also used to check or validate results obtained by more sophisticated flexible methods.

The rigid method assumes that a thick slab is infinitely rigid when compared to the soil, and hence the flexural deflections are negligible and do not influence the contact pressure, which is assumed to vary linearly as a result of simultaneous rigid body translation and rotation of the raft. Closed-form solutions to estimate the contact pressure underneath a rigid eccentrically loaded circular raft can be found elsewhere (e.g., Teng 1975). For a rigid rectangular raft with area $B \times L$, the contact pressure $q$ at any point, with coordinates $x$ and $y$ with respect to a Cartesian coordinate system passing through the centroid of the raft area (see Figure 3.22), is given by

$$q(x, y) = \frac{Q_t}{B \times L} \pm \frac{M_x}{I_x} y \pm \frac{M_y}{I_y} x \quad (3.101)$$

where
Figure 3.21: Probabilistic design charts (after Sivakugan and Johnson 2004).
$Q_t = \sum_{i=1}^{n} Q_i = \text{total load on the raft (sum of all column loads)}$

$M_x = Q_t e_y = \text{moment of the column loads about the x-axis}$

$M_y = Q_t e_x = \text{moment of the column loads about the y-axis}$

$e_x, e_y = \text{eccentricities about the x- and y-axes, respectively}$

$I_x = BL^3/12 = \text{moment of inertia about the x-axis}$

$I_y = BL^3/12 = \text{moment of inertia about the y-axis}$

The contact pressure distribution given by Equation 3.101 is used to estimate raft settlement, bearing capacity, bending moment, and shear forces. Static equilibrium in the vertical direction causes the resultant of column loads $Q_t$ to be equal and opposite to the resultant load.
obtained from integration of the reactive contact pressure in Equation 3.101. For simplicity, the rigid method assumes that the raft is analyzed by tributary areas in each of two perpendicular directions, similar to the structural design of two-way flat slabs, as shown by the shaded areas in Figure 3.22. To calculate bending moments and shear forces, each of two perpendicular bands is assumed to represent independent continuous beams under a constant average upward pressure \( q_{av} \) estimated by Equation 3.101. This simplification violates equilibrium because bending moments and shear forces at the common edge between adjacent bands are neglected. Therefore, the contact pressure \( q_c \) obtained by dividing the sum of the column loads in each band by the total area of the band is not equal to \( q_{av} \) computed from Equation 3.101. Hence, one of the following two assumptions is made in practice to estimate bending moments and shear forces in the assumed independent beams shown in Figure 3.22: (1) the actual column loads \( Q_{ij} \) are multiplied by an adjustment factor \( \mu \) to make the contact pressure \( q_c \) equal to \( q_{av} \) (Das 1984) or (2) the columns are assumed to be rigid supports whose reactions \( R_{ij} \) are calculated assuming uniform contact pressure \( q_{av} \). The above simplifications produce adjusted loads (\( \mu Q_{ij} \) with assumption 1) or rigid support reactions (\( R_{ij} \) with assumption 2) that are not equal to the corresponding column loads. This error is tolerated in practice provided the raft is regarded as rigid, which satisfies the following requirements (American Concrete Institute 1988):

1. The column loads and column spaces do not vary from each other by more than 20%.
2. The spacing \( (l) \) between column loads is such that

\[
b \leq \frac{1.75}{4} \sqrt{\frac{k_s l}{4E_f I_f}}
\]

(3.102)

where \( l \) is the width of the band, \( E_f \) is the modulus of elasticity of the foundation material, \( I_f \) is the moment of inertia of the cross section of the equivalent continuous beam, and \( k_s \) is the coefficient of the subgrade reaction defined as

\[
k_s = \frac{q}{\delta}
\]

(3.103)

where \( \delta \) is the settlement produced by a gross bearing pressure \( q \). \( k_s \) is measured in pressure per unit of length, sometimes referred as force per cubic length, which should not be confused with the soil unit weight. If the above requirements are not met, the raft should be designed as a flexible raft. In addition to calculation of bending moments and shear forces, the punching shear under each column also must be checked.

Flexible methods are based on analytical linear elastic solutions (Milović 1992; Hemsley 1998) and numerical solutions such as the method of finite differences and method of finite elements, where the stiffness of both the soil and structural members can be taken into account. Early flexible numerical methods were based on the numerical solution of the fourth-order differential equation governing the flexural behavior of a plate by the method of finite differences. The raft is treated as a linear elastic structural element whose soil reaction is replaced by an infinite number of independent linear elastic springs, following the Winkler
hypothesis. The soil elastic constant is given by the coefficient of the subgrade reaction \( k_s \) defined by Equation 3.103.

Let’s consider an infinitely long beam of width \( b \) (m) and thickness \( h \) (m) resting on the ground and subjected to a point load where the soil reaction is \( q^* \) (kN/m) at distance \( x \) from the origin. From the principles of engineering mechanics, it can be shown that the bending moment at distance \( x \) is

\[
M = E_{\text{raft}} I_{\text{raft}} \frac{d^2 z}{dx^2}
\]

The shear force at distance \( x \) is

\[
V = \frac{dM}{dx} = E_{\text{raft}} I_{\text{raft}} \frac{d^3 z}{dx^3}
\]

The soil reaction at distance \( x \) is

\[
q^* = \frac{dV}{dx} = E_{\text{raft}} I_{\text{raft}} \frac{d^4 z}{dx^4} = -zk'
\]

Here, \( I_{\text{raft}} \) is the moment of inertia of the cross section of the beam about the bending axis, given by \( bh^3/12 \), and \( k' \) is the subgrade reaction of the Winkler beam (in kN/m²), which is related to \( k_s \) by:

\[
k' = k_s b
\]

Therefore, Equation 3.106 becomes:

\[
E_{\text{raft}} I_{\text{raft}} \frac{d^4 z}{dx^4} = -zk_s b
\]

Equation 3.108 can be solved with appropriate boundary conditions, and the deflections of the Winkler beam on elastic springs can be obtained.

Approximate methods to estimate \( k_s \) as a function of the soil and foundation material elastic constants can be found elsewhere (e.g., Teng 1975; Bowles 1996; Das 2007; Coduto 2001; Lopes 2000). A major limitation of the early flexible methods comes from the unrealistic estimates of the coefficient of the subgrade reaction. The difficulty in estimating \( k_s \) comes from the fact that it is not a fundamental soil property, and its magnitude depends on factors such as the (1) width of the loaded area, (2) shape of the loaded area, (3) depth of the loaded area, (4) location on the raft for which settlement is being considered, and (5) time. Thus, considerable judgment and personal experience are required to select appropriate \( k_s \) values for design purposes. A practical way to estimate \( k_s \) roughly is to calculate the settlement \( \delta \) by any method outlined in Sections 3.5 and 3.6 and back calculate \( k_s \) by Equation 3.103.

The modulus of the subgrade reaction can be obtained from a plate load test, typically using a 300-mm square plate. Typical values of \( k_{0.3} \) (from a 300-mm plate) are given in Table 3.5. In granular soils, the value of \( k_s \) for a \( B \times L \) rectangular footing is given by:
In cohesive soils:

\[
k_s = k_{0.3} \left( \frac{0.3}{B} \right) \left( \frac{1 + 0.5B/L}{1.5} \right)
\]

(3.110)

Vesic (1961) suggested that \( k_s \) can be estimated from Equation 111 by:

\[
k_s = 0.65 \frac{E_s}{B(1 - \nu^2)} \sqrt{\frac{E_s B^4}{E_F I_F}}
\]

(3.111)

Here, \( E_s \) = Young’s modulus of the soil, \( E_F \) = Young’s modulus of the footing, \( I_F \) = moment of inertia of the foundation’s cross section, and \( \nu \) = Poisson’s ratio of the soil.

Flexible methods based on the coefficient of the subgrade reaction generally are not suitable for reliable estimates of total settlement, although such methods may provide acceptable estimates of differential settlement. Since bending moments and shear forces generally are not very sensitive to variations in \( k_s \), flexible methods are still widely used for structural raft design.

Due to the increasing popularity of efficient, user-friendly computational codes for geotechnical design based on finite elements and advanced versions of the finite difference method, it was possible to overcome some of the limitations of the rigid method and early flexible methods, such as equilibrium and compatibility requirements, irregular soil layers, nonlinear inelastic soil response, three-dimensional modeling, soil-structure interaction, coupled flow-deformation analyses, and dynamic loading. Three-dimensional numerical modeling of a soil-structure interaction problem is still a time-consuming task, and hence numerical modeling commonly is limited to simpler two-dimensional analyses whenever feasible. The finite element method and advanced versions of the finite difference method allow different possibilities for modeling the soil stiffness, including linear elastic analysis, hyperbolic models, and elastic-plastic and viscous-plastic models (e.g., Duncan and Chang 1970; Chen and Saleeb 1982; Desai and Sirwardena 1984). Although more refined elastic-plastic and viscous-plastic models represent a better idealization of the soil response, simpler models such as linear elastic analyses limited by a simple failure criterion (e.g., Mohr-Coulomb), hyperbolic models, and

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Soil Condition</th>
<th>( k_{0.3} ) (MN/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Granular soils</td>
<td>Loose</td>
<td>10–25</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>25–125</td>
</tr>
<tr>
<td></td>
<td>Dense</td>
<td>125–250</td>
</tr>
<tr>
<td>Dry or moist</td>
<td>10–15</td>
<td>25–40</td>
</tr>
<tr>
<td></td>
<td>125–150</td>
<td>125–150</td>
</tr>
<tr>
<td>Saturated</td>
<td>Stiff</td>
<td>10–20</td>
</tr>
<tr>
<td></td>
<td>Very stiff</td>
<td>20–40</td>
</tr>
<tr>
<td></td>
<td>Hard</td>
<td>40+</td>
</tr>
<tr>
<td>Cohesive soils</td>
<td>Stiff</td>
<td>10–20</td>
</tr>
<tr>
<td></td>
<td>Very stiff</td>
<td>20–40</td>
</tr>
<tr>
<td></td>
<td>Hard</td>
<td>40+</td>
</tr>
</tbody>
</table>
the Cam-Clay elastic-plastic model probably are the most widely used in practice due to their simplicity, ease in estimating input parameters, computational speed, and numerical stability.

3.7.2 Bearing Capacity and Settlement of Rafts

Raft bearing capacity and settlement calculations generally follow the same conventional methods outlined previously for pad footings. Although numerical analyses are gaining increased acceptance, conventional methods still are widely used in practice, either in preliminary estimates or to cross-check numerical calculations. Numerical methods rely on input soil constants estimated mostly by laboratory tests. Apart from the differences between field and laboratory soil responses, actual stress path dependency in the field may not be adequately reproduced by simple paths simulated in the laboratory. This is especially true for the simpler numerical models more commonly used in practice, as they rely on a reduced number of soil constants, which may not thoroughly simulate the complex soil behavior. Hence, some adjustments in the input soil constants may be needed in numerical analyses, and conventional methods may be very helpful in validating numerical results.

An increased raft area (and a corresponding decreased contact pressure) generally can increase the raft bearing capacity. However, increasing the raft area may not prevent bearing capacity failure in cases of low-strength soils underneath a raft, including cases where the raft bears on firm deposits underlain by soft sediments. For clays under undrained conditions ($\phi_u = 0$), $N_u = 0$, and hence the ultimate bearing capacity becomes independent of the foundation width $B$. Increasing the raft area also may be ineffective in reducing total settlement because a larger foundation width also will encompass a deeper volume of the deformable soil mass. Therefore, increasing the raft area to reduce total settlement or to increase bearing capacity in weak soils may be costly and inefficient. Total settlement can be minimized by a larger foundation area in cases where the compressible soil stratum is at a relatively shallow depth (hence part of the contact pressure can be distributed to a more resistant soil layer beneath the weak soil) or when the soil stiffness increases significantly with depth.

An efficient way to increase bearing capacity and decrease total settlement is to design floating (or compensated) rafts whereby the total weight of construction is compensated for by previous excavation of the same or a slightly higher total weight of soil and water (Golder 1975; Zeevaert 1983). This leads to a higher gross ultimate bearing capacity as a result of the deeper raft depth $D_f$ and lower total settlement because the soil elements beneath the raft become overconsolidated. Fully compensated floating rafts in normally consolidated deposits settle less than noncompensated or partially compensated rafts, where loading reaches the virgin compression and produces undesired long-term deformations.

Excavation unloading causes bottom heave that may progress to bottom failure. Zeevaert (1983) classifies bottom heave as P-heave (plastic heave, ultimately leading to bottom failure), E-heave (elastic heave, caused by nearly instantaneous elastic unloading and further upward relief by seepage pressure), S-heave (swelling heave, time-dependent upward displacement at constant total stress), and D-heave (driving heave, nearly instantaneous upward movement caused by the soil displaced by pile driving in the case of piled rafts).

Figure 3.23 shows the unloading path OBC of a fully unloaded normally consolidated soil element located right beneath the bottom of the excavation. Excavation unloading causes an upward displacement corresponding to ABC, where AB is nearly instantaneous E-heave and BC is time-dependent S-heave. Time-dependent S-heave depends on the amount of time the excavation remains open, and hence construction reloading should proceed as quickly as
possible to minimize BC. For settlement calculations, S-heave should be measured soon after the bottom of the excavation is reached and added to settlements estimated by the recompression index $C_r$ (FQ). For preliminary estimates of settlements produced by E- and S-heave recompression, the recompression index $C_{hr1}$ (CO) may be selected for soil elements close to the excavation bottom. $C_{hr1}$ depends on the duration the sample is exposed to relaxation until it is recompressed in the laboratory. Hence, $C_{hr1}$ is higher than the recompression index $C_r$ (FQ), selected for deeper soil elements not affected by S-heave. Intermediate values $C_{hr2}$ (EP) between $C_{hr1}$ and $C_r$ may be selected at intermediate depths, where unloading may produce smaller S-heave (DE). D-heave is minimized in piled rafts by an alternating driving program, where a sufficient distance is allowed between piles during driving. Next, intermediate piles are driven between two previously driven piles only after a prescribed time has elapsed. D-heave also is minimized by driving from the center toward the edges of the raft. Piles can minimize E- and S-heave as long as they are driven from the surface, before excavation commences. This allows mobilization of negative skin friction along the pile right from the early stages of excavation. Upon construction reloading, rafts with full friction piles are efficient for controlling differential settlements and building tilting induced by eccentric loading, uneven primary and secondary consolidation settlements, neighboring construction, dewatering, and deformation of wall supports. Design criteria for piled rafts can be found elsewhere (e.g., Zeevaert 1983; Franke et al. 2000; Katzenbach et al. 2000; Poulos 2000).

Compensated nonpiled rafts are efficient for ensuring adequate bearing capacity and tolerable settlements only when not influenced by unforeseen features in the compressible strata and from adjacent existing or new buildings. Massad (2005) explains the excessive tilting of some buildings in the city of Santos, Brazil, as due to local overconsolidation produced by mobile sand dunes. Figure 3.24 shows possible types of damage caused by adjacent construction (Teixeira 2003). In Figure 3.24a, buildings A and B are constructed at approximately the
same time. In Figure 3.24b, buildings A and B are constructed at approximately the same time and building C shortly thereafter. In Figure 3.24c, building B is constructed long after building A. In Figure 3.24d, buildings B and C are constructed nearly simultaneously after existing building C. The interaction with adjacent construction always should be carefully investigated in foundation design. Excavation and subsequent building construction always must rely on rigorous and permanent monitoring of the new building and all adjacent buildings for mutual settlement and tilting.

### 3.8 Shallow Foundations under Tensile Loading

Tensioned foundations are common in civil engineering applications such as transmission towers, harbors, basement slabs under pressure, industrial equipment, etc. Procedures for the design of tensioned foundations are discussed in this section, including specific recommendations for the more common transmission tower foundations. Starting with a distinction between shallow and deep modes of failure, this section presents the most common failure mechanisms for shallow failure under tension and procedures for calculation of the foundation tensile capacity under vertical and inclined loading. Emphasis is given to the influence of the strength of the compacted backfill compared to the strength of the natural soil. The design considerations presented here are the results of three decades of research carried out at the Federal University of Rio de Janeiro since the 1970s, based on many full-scale tensile tests on different types of transmission tower foundations in several soil formations through-
Design of Shallow Foundations

out Brazil (Barata et al. 1978, 1979, 1985; Danziger 1983; Danziger et al. 1989; Pereira Pinto 1985; Ruffier dos Santos 1985, 1999; Garcia 2005, Danziger et al. 2006). The practical recommendations are based on the second author’s experience in the design and construction of foundations for extra-high-voltage transmission lines over the last 30 years across Brazil, including very long transmission systems in the Amazon region and the Itaipu 750-kV transmission system. The criteria for predicting tensioned foundation capacity as discussed in this session are based mainly on the comprehensive work developed at the University of Grenoble (e.g., Martin 1966; Biarez and Barraud 1968; Martin 1973), due to its wide applicability for different types of soils, failure modes, load inclinations, and embedment depths and its good agreement with several full-scale tests on different types of foundations in a wide variety of soils.

In tensile foundations, shear strains are more pronounced than volumetric strains in contributing to displacement. In foundations under compressive loads, especially in weak soils, volumetric strains are predominant in contributing to settlement. As a result, tensioned foundations generally produce smaller displacements compared to foundations under the same compressive load in the same type of soil. Therefore, the design of foundations under tensile loads is conceived under limit equilibrium criteria in most cases, in contrast to compressed foundations, where consideration of limit equilibrium and settlement is important. Further discussion on prediction of displacement of tensioned foundations is provided by Trautmann and Kulhawy (1988) and Sakai and Tanaka (2007). Finite element analyses also are useful for predicting displacement of tensioned foundations, although more accurate three-dimensional simulations may be time consuming for design purposes. In this section, the design recommendations are restricted to limit equilibrium analyses.

### 3.8.1 Tensile Loads and Failure Modes

Tensioned foundations can be subjected to permanent as well as transient loading. In the case of transmission lines, permanent loading is caused by angle and anchor loading in the towers. Angle loading occurs when there is a change in the direction of the transmission line at the tower. Anchor loading occurs on one side of the first and last tower in a row of towers (called end-of-line or anchor towers), resulting in unbalanced forces at the sides of the towers, produced by different cable tension and construction load. For design purposes, temperature variation in the conductors also may be regarded as permanent loading. Transient loading occurs due to wind load (usually the dominant design load) and sudden mechanical failure of the conductors.

Self-supported transmission towers (Figure 3.25a) can apply alternate concentric compression/tension loads (Figures 3.26a and 3.26b) or eccentric loads (Figure 3.26c) to the foundation. Guyed towers (Figure 3.25b) transmit concentric orthogonal tension loads to the inclined guy foundation and compressive eccentric loads to the central mast foundation. For the typical design and inclination of a tower guy (about 30° to 35° to vertical), the effect of load inclination should be accounted for in the foundation design, as the ultimate tensile capacity is dependent on the load/plate inclination.

The foundation design loads usually are provided by the tower manufacturer. The foundation loads are calculated under different load hypotheses. For self-supported towers (Figure 3.25a), the design loads are given by superposition of the vertical (tension/compression) and two mutually perpendicular horizontal loads that act transversely and along the transmission line. The foundation designer takes into account the most unfavorable load hypothesis for
each foundation element, making a clear distinction between permanent and transient loading. A safety factor of 3 for permanent loads and 2 for transient loads, with respect to the theoretical ultimate tensile capacity, generally is recommended for tensioned foundations. Intermediate values may be used for simultaneous permanent and transient loading.

Steel grillage foundations (Figures 3.26a) or footings with inclined pedestals (Figure 3.26b) for self-supported transmission towers are subjected to a resultant tension/compression load that is approximately in the same direction as the tower leg, thus transmitting mostly concentric loading to the foundation. Moreover, the usual slope of a typical self-supported tower leg is small. Thus, for practical purposes, the tensile capacity of steel grillage foundations or footings with inclined pedestals for self-supported towers is calculated for vertical loading only, neglecting the secondary effects of load inclination and minor eccentricities. In contrast, self-supported towers on footings with a vertical pedestal (Figure 3.26c) introduce eccentricities in two orthogonal directions, parallel and perpendicular to the direction of the transmis-

FIGURE 3.25 Most common types of towers: (a) self-supported tower and (b) guyed tower.

FIGURE 3.26 Common foundations for self-supported towers: (a) steel grillage, (b) footing with inclined pedestal, and (c) footing with vertical pedestal.
The behavior of tensioned foundations under an eccentric oblique load was studied by Meyerhof (1973a, 1973b). A simplified procedure for practical design is to determine the equivalent reduced foundation dimensions to account for the double eccentricity, similar to the design of compressive eccentric loads discussed in Section 3.3. Horizontal loading and foundation eccentricity may play a dominant role in the design of piled transmission tower foundations on weak soils. There are situations in practice where foundation pedestals need to be high, such as in cases of significant seasonal variation in the flooding level of rivers or inundated areas crossed by transmission lines. High pedestals are much easier to build vertically than inclined. The overturning moments generated in such cases may be very high, and the corresponding footing dimensions would be very large. Hence, the use of prestressed anchors at the foundation corners is generally a cost-effective way to absorb high overturning moments (Danziger et al. 2006).

Vertical or nearly vertical tensioned plates can fail in shallow and deep modes of failure (Martin 1966; Biarez and Barraud 1968; Meyerhof and Adams 1968; Martin 1973), as shown in Figure 3.27a for firm soils and Figure 3.27b for weak soils. In the shallow mode (Figure 3.27a.1 and b.1), the failure surface reaches the ground level, and all applied tensile load is resisted by the plate. Thus, the shaft (or pedestal) transmits the applied tensile load directly from the structural member to the plate. In the deep mode (Figure 3.27a.2 and b.3), the tensile load is shared by the plate and the shaft where the failure surface around the plate does not reach the ground level. Therefore, the applied tensile load is not entirely transmitted to the

**FIGURE 3.27** Shallow and deep failure modes: (a) firm soils where $\alpha < 0$ and (b) weak soils where $\alpha > 0$. (a.1) Shallow mode in firm soils, (a.2) deep mode in firm soils, (b.1) shallow mode in weak soils, (b.2) intermediate mode in weak soils, and (b.3) deep mode in weak soils (Biarez and Barraud 1968; Martin and Cochard 1973).
plate. The ultimate tensile load $Q_{\text{ult}}$ obtained as a function of the plate depth in the shallow and deep modes is shown qualitatively in Figure 3.28. The dashed and solid lines represent the shallow and deep modes, respectively (Biarez and Barraud 1968; Martin and Cochard 1973). The two curves intersect at the critical depth $D_c$, where the failure mode changes from shallow to deep or vice versa. To determine whether failure will be in the shallow or deep mode, the calculations for both modes should be performed and the one that corresponds to the smaller tensile resistance chosen. However, full-scale load tests indicate that the critical depth usually is less than two to three times the diameter of a circular plate or the width of a square plate. Therefore, for typical depths and dimensions of ordinary shallow foundations used in transmission towers, failure would be in the shallow mode. Thus, the behavior of foundations under tensile loading discussed in this section is limited to the shallow mode.

The shapes of the failure surface in the shallow and deep modes are dictated by the type of soil in which the foundation is placed (Biarez and Barraud 1968) and by the inclination of the tensile load (Martin and Cochard 1973). The simplified shallow mode shown in Figure 3.29 was developed for homogeneous firm soils under vertical loading. The actual curvilinear failure surface observed in tensile tests is replaced by an equivalent simplified conical surface.

**Figure 3.28** Determination of the critical depth (Biarez and Barraud 1968; Martin and Cochard 1973).

**Figure 3.29** Observed (dashed line) and simplified (solid line) failure modes in firm soils (Biarez and Barraud 1968).
with a slope $\alpha$, as indicated in Figures 3.27 and 3.29. The shape of the failure surface (and hence slope $\alpha$) depends on the type of soil and the friction angle $\phi$, as shown in Figure 3.27. For shallow plates, Biarez and Barraud (1968) and Martin and Cochard (1973) conceived three cases of distinct failure modes, depending on the soil type:

1. Granular soils (dense or loose), where the failure surface develops outward with an average inclination $\alpha = -\phi$
2. Firm clayey soils with $\phi > 15^\circ$, where the failure surface develops outward with an average inclination $\alpha = -\phi/4$
3. Soft clayey soils with $\phi < 15^\circ$, where the failure surface develops inward with an average inclination $\alpha = \tan^{-1}(0.2)$

The convention used here is that $\alpha < 0$ for a failure surface that propagates outward from the plate and $\alpha > 0$ when this surface propagates inward. The above failure modes have been observed in model tests in homogeneous soils in the laboratory and often have been confirmed by full-scale tests. The shallow mode for weak soils such as saturated soft clays (case 3) generally is of little importance in practice, since in this case the weak soil above the foundation almost always is replaced by more resistant, preselected compacted backfills, as in Figure 3.30. In situations such as Figure 3.30a, the undrained tensile capacity of the plate is estimated simply as $c_u p_b D$, where $c_u = \text{undrained shear strength of the clay}$, $p_b = \text{perimeter of the plate}$, and $D = \text{plate depth}$. In situations such as Figure 3.30b, the tensile capacity is estimated either as in case 1 or case 2.

The failure modes shown in Figures 3.27 and 3.29 are applicable to homogeneous soils. Sakai and Tanaka (2007) investigated the tensile capacity of layered soils. To account for the inhomogeneity introduced by the compacted backfill, the tensile capacity is controlled by the weaker of the two materials: backfill or surrounding natural soil. If the backfill is weaker than the natural soil, the failure takes place at the vertical interface ($\alpha = 0$). If the natural soil is weaker, the failure takes places within the natural soil, with the conical failure surface propagating outward from the plate ($\alpha = -\phi/4$ or $\alpha = -\phi$).

The effect of load inclination $\beta$ (with respect to the vertical direction) in homogeneous soils is shown in Figure 3.31a for the shallow mode of failure and in Figure 3.31b for the deep mode (Martin and Cochard 1973). In the shallow mode, however, depending on the load inclination and the relative strength of the compacted backfill with respect to the natural soil, the actual shallow failure mode is likely to depart from the idealized modes shown in Figure

**FIGURE 3.30** Uplift capacity in soft clays: (a) vertical excavation and (b) sloped excavation.
Figure 3.31 Failure modes for inclined load in firm soils: (a.1) vertical load in shallow mode, (a.2) inclined load in shallow mode, (a.3) horizontal load in shallow mode, (b.1) vertical load in deep mode, (b.2) inclined load in deep mode, and (b.3) horizontal load in deep mode.

3.3.1a and produce distinct failure angles $\alpha_L$ and $\alpha_R$ (at the left and right edges of the plate), as in Figure 3.32. The failure modes for shallow inclined plates at moderate load inclination ($\beta < 30^\circ$), as in the case of guyed transmission towers shown in Figure 3.25b, are similar to the ones for horizontal plates under uplift loading (Martin and Cochard 1973). For steeper inclinations ($\beta > 30^\circ$), the failure modes change as the angle $\beta$ increases (Figure 3.31b). In all the models discussed below, it is assumed that the load is acting normal to the plate.

3.8.2 Tensile Capacity Equations in Homogeneous Soils: Grenoble Model (Martin and Cochard 1973)

3.8.2.1 Moderately Inclined Plates ($\beta < 30^\circ$), Including Horizontal Plates ($\beta = 0^\circ$)

As with most methods discussed in the literature, the uplift capacity $Q_{\text{ult}}$ of plates installed at a shallow depth can be expressed by tensile capacity factors, similar to the bearing capacity formulae, as (Biarez and Barraud 1968; Martin and Cochard 1973)

$$Q_{\text{ult}} = p_b \frac{D}{\cos \beta} \left[ cM_c + \frac{D}{\cos \beta} (M_{\phi} + M_{\gamma}) + q_0M_{\gamma} \right] + \gamma S_b D + W \cos \beta$$

(3.112)
where $D$ = depth, $p_b$ = plate perimeter, $S_b$ = plate area, $c$ = cohesion, $\gamma$ = unit weight of the soil, $W$ = foundation self-weight, and $q_0$ = external surcharge acting at the ground level. $M_c$, $(M_\phi + M_\gamma)$, and $M_\eta$ are dimensionless tensile capacity factors dependent on the soil type and friction angle $\phi$, calculated by the set of formulae given in Appendix A. The term

$$
\left[ cM_c + \frac{D}{\cos \beta} (M_\phi + M_\gamma) + q_0 M_\eta \right]
$$

in Equation 3.112 accounts for the average shear stress acting on the failure surface and is usually the dominant term in the foundation tensile capacity. In the absence of external surcharge at the soil surface (the most common situation in practice), the term $q_0 M_\eta$ vanishes. The term $W$, the foundation self-weight, is negligible in the case of steel grillage foundations. The term $\gamma S_b D$ accounts for the weight of the soil above the plate.

### 3.8.2.2 Steeply Inclined Plates

The following applies to a steeply inclined ($\beta > 30^\circ$) shallow rectangular plate under a concentric load acting normal to the plate:

$$
Q_{ult} = BL(cN_c + 0.5B\gamma N_\phi + q_0 N_\eta) + W \cos \beta
$$

(3.113)

where $B$ is the width and $L$ is the length of a rectangular plate. In the case of a circular plate, the plate area is calculated assuming an equivalent radius $R_e = (B + L)/\pi$. The term $(cN_c + 0.5B\gamma N_\phi + q_0 N_\eta)$ accounts for the average shear stress acting on the failure surface and usually
is the dominant term in the foundation tensile capacity. The tensile capacity factors $N_c$, $N_\phi$, and $N_q$ are given by the set of formulae in Appendix B. For load inclinations close to the limit $\beta = 30^\circ$, it is advisable to calculate the tensile capacity separately by Equations 3.112 and 3.113 and choose the smaller value. The tensile capacity factors applicable to Equations 3.112 and 3.113 are obtained easily with spreadsheets or programmable hand calculators.

**Example 1.** Determine the uplift capacity of a 2.50-m x 2.50-m horizontal square grillage embedded $D = 2.30$ m. The soil strength parameters are $c = 15$ kPa and $\phi = 25^\circ$, and the unit weight is $\gamma = 16$ kN/m$^3$. The backfill is assumed to be stronger than the natural soil.

In this example, the strength parameters of the more resistant backfill are not needed in the calculations, since failure is expected to develop through the natural soil ($\alpha = -\phi/4$). The uplift capacity factors applicable to Equation 3.12 are obtained from Appendix A as $M_c = 0.83$ and $M_\phi + M_\gamma = 0.22$. Neglecting the weight of the grillage and assuming no surcharge at the soil surface ($W = 0$ and $q_0 = 0$), the uplift capacity is calculated from Equation 3.12 taking $\beta = 0$ (vertical loading) as:

$$Q_{ult} = 4 \times 2.50 \times 2.30 \left[ 15.0 \times 0.83 + 16.0 \times 2.30 \times 0.22 \right] + 2.50^2 \times 2.30 \times 16.0$$

$$Q_{ult} = 472.6 + 230.0 = 702.6 \text{ kN}$$

**Example 2.** For the same horizontal grillage as in example 1, now the backfill is less resistant than the natural soil. The backfill strength parameters are $c = 15$ kPa and $\phi = 25^\circ$. The backfill unit weight is $\gamma = 16$ kN/m$^3$.

In this example, the strength parameters of the more resistant natural soil are not needed in the calculations, since failure is expected to develop at the interface of the backfill with the natural soil ($\alpha = 0$), controlled by the strength of the backfill. From Appendix A, the tensile capacity factors are calculated as $M_c = 0.66$, $M_\phi + M_\gamma = 0.19$, and $M_q = 0.31$. Therefore, from Equation 3.12:

$$Q_{ult} = 4 \times 2.50 \times 2.30 \left[ 15.0 \times 0.66 + 16.0 \times 2.30 \times 0.19 \right] + 2.50^2 \times 2.30 \times 16.0$$

$$Q_{ult} = 388.5 + 230.0 = 618.5 \text{ kN}$$

The above examples show that for the same strength parameters in both cases, the uplift capacity in example 1 (failure through the less resistant natural soil) is about 13% higher than in example 2 (failure through the interface). This illustrates the need for adequate compaction of the backfill, as a poorly compacted backfill significantly decreases the overall tensile capacity.

**Example 3.** Determine the tensile capacity of a rectangular plate ($B = 0.5$ m and $L = 1.35$ m), inclined $\beta = 33.5^\circ$, embedded $D = 1.28$ m, and loaded normally to the plate. The soil strength parameters are $c = 9$ kPa and $\phi = 23^\circ$, and the soil unit weight is $\gamma = 13.4$ kN/m$^3$.

Assuming initially that the plate is at moderate inclination, it follows from Appendix A (for $\phi = 23^\circ$ and $\alpha = -23/4 = -5.75^\circ$) that $M_c = 0.897$, $M_\phi + M_\gamma = 0.21$, and $M_q = 0.267$. Neglecting the plate self-weight and assuming no surcharge at the soil surface ($q_0 = 0$ and $W = 0$), then:
\[
Q_{\text{ult}} = 2x(0.5 + 1.35)x \frac{1.28}{\cos 33.5^\circ} \left[ 9x0.897 + 18.4x \frac{1.28}{\cos 33.5^\circ} x0.21 \right] \\
+ 0.5x1.35x1.28x18.4 \\
Q_{\text{ult}} = 79.5 + 15.9 = 95.4 \text{ kN}
\]

**Example 4.** Assuming in example 3 that the plate inclination is steep, it follows from Appendix B (for \(\phi = 23^\circ\)) that \(N_c = 9.523, N_\phi = 13.563, \text{ and } N_q = 4.042\). Therefore, taking \(q_0 = 0\) and \(W = 0\):

\[
Q_{\text{ult}} = 0.5x1.35x(9x9.523 + 0.5x18.4x0.5x13.563) = 99.96 \text{ kN}
\]

Comparing examples 3 and 4, the smaller value \(Q_{\text{ult}} = 95.4 \text{ kN}\) (plate at moderate inclination) should be taken as the plate tensile capacity.

Figure 3.32 shows that the failure surface propagates through the natural soil and compacted backfill in the case of an inclined foundation. Thus, good engineering judgment is required to select the strength parameters to be used in the design of inclined foundations in practice, due to the influence of the compacted backfill. For safe design, however, it is recommended that the strength parameters from the smaller values corresponding to either the natural soil or the compacted backfill be chosen. As with vertical tensioned foundations, proper backfill compaction is essential for adequate foundation performance.

**Appendix A**

**Tensile Capacity Factors for Shallow Plates at Moderate Load Inclination \((\beta < 30^\circ)\) or Horizontal Plates \((\beta = 0)\): Grenoble Model \((\text{Martin and Cochard 1973})\)**

\[
Q_{\text{ult}} = F_b \frac{D}{\cos \beta} \left[ cM_c + \frac{\gamma D}{\cos \beta} (M_\phi + M_\gamma) + q_0M_q \right] \\
+ \gamma S_bD + W \cos \beta
\]

\(M_c, (M_\phi + M_\gamma), \text{ and } M_q\) are dimensionless tensile capacity factors dependent on the friction angle \(\phi\) and calculated by the following set of formulae:

\[
M_c = M_{\text{c0}} \left[ 1 - \frac{\tan \alpha}{2} \frac{D}{R} \frac{1}{\cos \beta} \right]
\]

\[
M_{\text{c0}} = - \frac{\tan \alpha}{\tan \phi} + \frac{f}{H} \cos \phi \left[ 1 + \frac{\tan \alpha}{\tan \phi} \right]
\]
\[
\frac{f}{H} = \tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right) \frac{\cos n - \sin \phi \cos m}{\cos n + \sin \phi \cos m}
\]

\[
m = -\frac{\pi}{4} + \frac{\phi}{2} + \alpha
\]

\[
\sin n = \sin \phi \sin m
\]

\[
M_\phi + M_\gamma = (M_{\phi 0} + M_{\gamma 0}) \left(1 - \frac{\tan \alpha}{3} \frac{D}{R} \frac{1}{\cos \beta}\right)
\]

\[
M_{\phi 0} + M_{\gamma 0} = \frac{\sin \phi \cos(\phi + 2\alpha)}{2 \cos^2 \alpha}
\]

\[
M_q = M_{q 0} \left(1 - \frac{\tan \alpha}{2} \frac{D}{R} \frac{1}{\cos \beta}\right)
\]

\[
M_{q 0} = M_{c 0} \tan \phi + \tan \alpha
\]

\(\beta\) is the load inclination to the vertical (which is zero for horizontal plates) and \(R\) is the radius of a circular plate or the equivalent radius of a rectangular plate with dimensions \(B \times L\), calculated as \(R = (B + L)/\pi\), except in the case of saturated clays, where \(R = (B + L)/4\). \(D\) is the plate depth, \(p_b\) is the plate perimeter, \(S_b\) is the plate area, \(c\) is the soil cohesion, \(\gamma\) is the unit weight of the soil, \(W\) is the self-weight of the foundation element, and \(q_0\) is the external surcharge acting at the ground level.

**Appendix B**

**Tensile Capacity Factors for Shallow Plates at Steep Load Inclination (\(\beta > 30^\circ\)): Grenoble Model (Martin and Cochard 1973)**

\[
Q_{ult} + BL(cN_c + 0.5B\gamma N_\phi + q_0 N_q) + W \cos \beta
\]

\(N_c, N_\phi,\) and \(N_q\) are dimensionless tensile capacity factors dependent on the friction angle \(\phi\) and calculated by the following set of formulae:

\[
N_\phi = A_\phi + B_\phi \left(\frac{D}{B} - \frac{1}{2} \sin \beta\right) + \left(C_\phi + \frac{B}{L} E_\phi\right) \left(\frac{D}{B} - \frac{1}{2} \sin \beta\right)^2
\]

\[
N_q = B_q + \left(C_q + \frac{B}{L} E_q\right) \left(\frac{D}{B} - \frac{1}{2} \sin \beta\right)
\]
\[ N_c = N_q \cot \phi \]
\[ A_\phi = b_o - p_o \]
\[ B_\phi = 2(b_o - p_o) \]
\[ C_\phi = 2 \sin \phi \left( \frac{b_o b_\phi}{l_2} + \frac{p_o p_\phi}{l_1} \right) \]
\[ E_\phi = 2 \sin \phi \left( b_o b_\phi + p_o p_\phi \right) \]
\[ b_o = \sin \beta \exp \left( -0.6 - 1.7\beta \right) \phi \]
\[ p_o = \sin(\beta - \phi) \exp \left( 2.45 + \frac{1}{\beta} - 0.8\beta \right) \phi \]
\[ b_\phi = \cos \beta \]
\[ p_\phi = 1.1 \cos \phi \]
\[ b_o = \frac{1 + \sin \phi}{1 - \sin \phi} \sqrt{\frac{1 - \sin^2 \phi \sin^2 \beta - \sin \phi \cos \beta}{1 - \sin^2 \phi \sin^2 \beta + \sin \phi \cos \beta}} \]
\[ p_o = \frac{1 - \sin \phi}{1 + \sin \phi} \exp \left\{ -\left[ \pi - 2\beta \tan \phi \right] \right\} \]
\[ b_\phi = \frac{\cos \phi}{1 - \sin \phi} \sqrt{\frac{2 - \sin^2 \phi \left( \cos \frac{\phi}{2} - \sin \frac{\phi}{2} \right)^2 - \sin \phi \left( \cos \frac{\phi}{2} - \sin \frac{\phi}{2} \right)}{\frac{2 - \sin^2 \phi \left( \cos \frac{\phi}{2} - \sin \frac{\phi}{2} \right)^2 + \sin \phi \left( \cos \frac{\phi}{2} - \sin \frac{\phi}{2} \right)}}} \]
\[ p_\phi = \frac{\cos \phi}{1 + \sin \phi} \exp \left[ \left( \frac{\pi}{2} - \phi \right) \tan \phi \right] \]

\[ l_1 = \tan \left( \frac{\pi}{4} - \frac{\phi}{2} \right) \exp - \left[ (\pi - 2\beta) \tan \phi \right] \]

\[ l_2 = \tan \left( \frac{\pi}{4} + \frac{\phi}{2} \right) \frac{\sin \left[ \left( \frac{\pi}{4} + \frac{\beta}{2} \right) - \left( \frac{\phi}{2} - \frac{n}{2} \right) \right]}{\sin \left[ \left( \frac{\pi}{4} + \frac{\beta}{2} \right) + \left( \frac{\phi}{2} - \frac{n}{2} \right) \right]} \]

\[ \sin n = \sin \beta \sin \phi \]

\( \beta (>30^\circ) \) is the load inclination to the vertical. \( B \) is the width and \( L \) is the length of a rectangular plate. In the case of a circular plate, the plate area is calculated assuming an equivalent radius \( R = (B + L)/\pi \). \( W \) is the self-weight of the foundation and \( q_0 \) is the external surcharge acting at the ground level.

**References**


